Q Form PDE by eliminating the arbitrary constants.

(i) $z = a(x+y) + b(x-y) + abt + c$

Solⁿ. $z = ax + ay + bx - by + abt + c$ —— (1)

Differentiating partially (1) w.r.t $x, y$ and $t$

$$\frac{\partial z}{\partial x} = a+b = p,$$

$$\frac{\partial z}{\partial y} = a-b = q, \quad \frac{\partial z}{\partial t} = ab$$

∵ we know

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$\Rightarrow p^2 - q^2 = 4\frac{\partial z}{\partial t}$$

Which is the reqd. PDE.

(ii) $(x-a)^2 + (y-b)^2 + z^2 = c^2$

Solⁿ. Differentiating partially (1) w.r.t $x$,

$$2(x-a) + 2z\frac{\partial z}{\partial x} = 0$$

$$\Rightarrow \cancel{2}(x-a) = -\cancel{2}zp \Rightarrow x-a = -zp$$

Differentiating partially (1) w.r.t $y$,

$$2(y-b) + 2z\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \cancel{2}(y-b) = -\cancel{2}zq \Rightarrow y-b = -zq$$

eq$^n$ (1) becomes,

$$(-zp)^2 + (-zq)^2 + z^2 = c^2$$

$$\Rightarrow z^2p^2 + z^2q^2 + z^2 = c^2$$

$$\Rightarrow z^2(p^2 + q^2 + 1) = c^2$$

which is the reqd. PDE A